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THE SUMMATION OF TWO SERIES.

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As the following series occur in the solution of problem 121, Calculus, the method of summing them, though neither difficult nor tedious, may be of interest to many of our readers.

$$y = 1 - \frac{n^2 x^2}{2!} + \frac{n^2(n^2+8)x^4}{4!} - \frac{n^2(n^4+40n^2+184)x^6}{6!} + \dots (A).$$

$$y = \frac{nx}{1!} - \frac{n(n^2+2)x^3}{3!} + \frac{n(n^4+20n^2+24)x^5}{5!} - \frac{n(n^6+70n^4+784n^2+720)x^7}{7!} + \dots (B).$$

The general term of both being

$$a_{r+4} = - \frac{r(r+1)_r + [2(r+2)^2 + n^2]a_{r+2}}{(r+3)(r+4)}.$$

$$\text{From (A), } dy/dx = -n^2 x + \frac{n^2(n^2+8)x^3}{3!} - \frac{n^2(n^4+40n^2+184)x^5}{5!} + \dots (1).$$

$$d^2 y/dx^2 = -n^2 + \frac{n^2(n^2+8)x^2}{2!} - \frac{n^2(n^4+40n^2+184)x^4}{4!} + \dots (2).$$

$$n^2 \text{ times (A) gives } n^2 y = n^2 - \frac{n^4 x^2}{2!} + \frac{n^4(n^2+8)x^4}{4!} - \dots (3).$$

$$2x(1+x^2) \text{ times (1) gives } 2x(1+x^2)(dy/dx) = -2n^2 x^2 + \frac{2n^2(n^2+2)x^4}{3!} - \dots (4).$$

$(1+x^2)^2$ times (2) gives

$$(1+x^2)^2 (d^2 y/dx^2) = -n^2 + \frac{n^2(n^2+4)x^2}{2!} - \frac{n^2(n^4+16n^2+16)x^4}{4!} + \dots (5).$$

$$(3) + (4) + (5) \text{ gives } (1+x^2)^2 (d^2 y/dx^2) + 2x(1+x^2)(dy/dx) + n^2 y = 0 \dots (6).$$

From (B), $dy/dx =$

$$n - \frac{n(n^2+2)x^2}{2!} + \frac{n(n^4+20n^2+24)x^4}{4!} - \frac{n(n^6+70n^4+784n^2+720)x^6}{6!} + \dots (7).$$

$$d^2 y/dx^2 = - \frac{n(n^2+2)x}{1!} + \frac{n(n^4+20n^2+24)x^3}{3!}$$

$$-\frac{n(n^6+70n^4+784n^2+720)x^5}{5!} \dots (8).$$

n^2 times (B) gives

$$n^2 y = \frac{n^3 x}{1!} - \frac{n^3(n^2+2)x^3}{3!} + \frac{n^3(n^4+20n^2+24)x^5}{5!} - \dots (9).$$

$2x(1+x^2)$ times (7) gives

$$2x(1+x^2)(dy/dx) = 2nx - \frac{2n^3 x^3}{2!} + \frac{3n^3(n^2+8)x^5}{4!} - \dots (10).$$

$(1+x^2)^2(d^2y/dx^2) =$

$$-\frac{n(n^2+2)x}{1!} + \frac{n^3(n^2+8)x^3}{3!} - \frac{n^3(n^4+30n^2+104)x^5}{5!} + \dots (11).$$

(9) + (10) + (11) gives $(1+x^2)^2(d^2y/dx^2) + 2x(1+x^2)(dy/dx) + n^2y = 0$,
the same as (6) ... (12).

Multiplying (6) and (12) through by $2(dy/dx)$, and integrating, we get
 $(1+x^2)^2(dy/dx)^2 + n^2y^2 + C = 0$.

When $x=0$, from (A), $y=1$, $dy/dx=0$.

When $x=0$, from (B), $y=0$, $dy/dx=n$.

\therefore In either case, $C = -n^2$. $\therefore (1+x^2)^2(dy/dx)^2 = n^2(1-y^2)$.

$$(1+x^2)dy/dx = \pm n\sqrt{1-y^2} \text{ or } \frac{dy}{\sqrt{1-y^2}} = \pm \frac{ndx}{1+x^2}.$$

$\therefore \sin^{-1}y = n \tan^{-1}x + D$, or $\sin^{-1}y = n \cot^{-1}x + D$.

From (A), when $x=0$, $y=1$.

$\therefore D = \frac{1}{2}\pi$. $\therefore \sin^{-1}y = n \tan^{-1}x + \frac{1}{2}\pi = n \cot^{-1}x + \frac{1}{2}\pi$.

$\therefore y = \sin(n \tan^{-1}x + \frac{1}{2}\pi) = \sin(n \cot^{-1}x + \frac{1}{2}\pi)$.

$\therefore y = \cos(n \tan^{-1}x) = \cos(n \cot^{-1}x)$.

From (B), when $x=0$, $y=0$.

$\therefore D = 0$. $\therefore \sin^{-1}y = n \tan^{-1}x = n \cot^{-1}x$.

$\therefore y = \sin(n \tan^{-1}x) = \sin(n \cot^{-1}x)$.